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Research Article

A Bi-objective Mathematical Model for the Patient Appointment Scheduling Problem in Outpatient Chemotherapy Clinics Using Fuzzy C-Means Clustering: A Case Study

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Abstract

In healthcare, the Patient Appointment Scheduling (PAS) problem is one of the critical issues in Outpatient Chemotherapy Clinics (OCC). In the wake of this, this paper proposes a novel bi-objective mathematical programming model for solving the PAS problem in OCC. The developed mathematical model is inspired by cellular manufacturing. The first objective function minimizes the completion time of all treatments, and the second objective function maximizes the use of nurses' skills while considering clustered patients about their characteristics. To solve the bi-objective mathematical model, for the first time a hybrid approach based on Torabi-Hassini (TH) and Lagrange method is utilized. The results indicate that an increase in the number of nurses will enhance the treatment completion speed and allocation of nurses' work skill. On the other hand, an increase in the number of chairs in clinics will decrease the assignments of nurses' skills priority. The study supports decision makers in considering nurses' skills for the PAS problem. The results also denote the desirability of the proposed model. To validate the proposed model, OCC in Tehran is considered as a case study. Computational results reveal that considering nurses' skills in OCC and using the fuzzy clustering model, as a new method in patient clustering, lead to achieving a desirable and more realistic outcome.

Keywords: Patient appointment scheduling; Mathematical programming model; Outpatient chemotherapy; Fuzzy clustering.

1. Introduction

A group of diseases in which abnormal cells begin to divide frequently uncontrollably is called cancer (Padmanabhan et al., 2017). Recently, cancer disease has increased in an alarming rate. In this regard, many scientists have sought to cure such a disease, which has increased about 50% in recent decade (APHA, 2016). For cancer treatment, several methods have been used including chemotherapy, radiotherapy, surgery, immunotherapy, and hormonal therapy. One of the major barriers in treating the cancer is patient's high mortality risk (Biagi et al., 2011). Regarding an increase in the cancer rate in all countries around the world, the demand for chemotherapy services has increased. Chemotherapy is used to control and treat cancers by utilizing drugs destroying cancer cells (Heshmat and Eltawil, 2018; Billaut et al., 2018; Prip et al., 2019)). Another reason for using chemotherapy is its good performance in the last decade (ACCR, 2014; NCI, 2017; Lai et al., 2018). The chemotherapy treatment schedules may depend on types of cancer, the targets of treatment, and the patient's state-level of health (Sevinc et al., 2013). Considering a large number of patients (with diverse characteristics), the patient appointment scheduling (PAS) problem plays a vital role in chemotherapy clinics (Heshmat et al., 2018). Some studies have used methods for determining the optimal regimen for chemotherapy in the treatment of patients in which the regimens are determined according to the characteristics of the patients (Swan, 1980).

Many researchers have focused on scheduling problems in chemotherapy clinics (Moreno and Blanco, 2018; Ahmadi-Javid et al., 2017). For solving the PAS problem, soft clustering (fuzzy clustering) along with mathematical programming has first been introduced by Heshmat et al. (2017). The use of hard clusters for clustering patient like K-means clustering by Heshmat et al. (2018) takes into account two aspects of the patient's features while all details and possible features of this disease must be considered. Whereas, in a soft clustering a membership function is considered for each data that specifies the amount of membership of each data to each cluster. For this purpose, in this paper, we use Fuzzy C-Means (FCM) for clustering. FCM is a method of clustering which allows one piece of data to belong to more than one cluster.
Solving the PAS problem, with the large volume of variables and data, might be time-consuming. So, using a proper solution approach which reduces computational time is necessary. In this paper, we use the Lagrange analysis method to reduce the computational time. Moreover, we studied a chemotherapy center in Tehran which provides chemotherapy services to cancer patients. This chemotherapy center includes many units, such as intensive care unit, treatment, diagnostic, care unit, among which the treatment unit has been considered in this study. Also, the treatment unit comprises several departments, such as surgery department, emergency department, oncology clinic, physiotherapy, radiotherapy, and outpatient chemotherapy, among which outpatient

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chemotherapy department has been considered for the case study. Figure 1 shows an overview of the outpatient chemotherapy and the scope of the study in this paper. As can be seen in Figure 1, the scope of the concerned problem in this paper relates to arriving the patients to the clinics and then their traveling to the treatment centers.

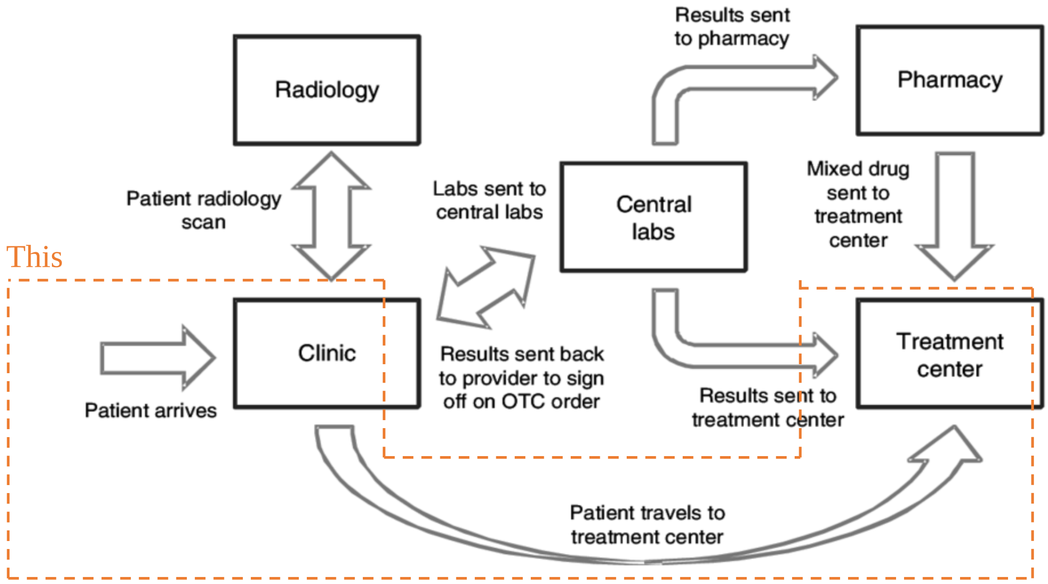

Fig. 1. Overview of outpatient chemotherapy.

The proposed model in this paper consists of four steps. The first step is related to identifying the characteristics of patients such as the type of cancer and treatment duration. The second step is relevant to patients clustering. In the clustering step, FCM clustering algorithm, which is a new method of clustering in this area, is used for finding the optimum cluster of patients containing characteristic of the type of cancer and treatment duration. Third and fourth steps are related to solving the problem's mathematical model. In this study, for the first time, TH method has been applied to convert a two-objective model to a single one. The use of this method makes the decision making based on the priority of each objective function possible for the treatment staff. In other words, we first identified the nurse's work skills, skill levels, and the necessity of each cluster to each skill. Then, we solved the proposed mathematical model using the Lagrange method. The rest of the paper is organized as follows: The literature review is presented in section 2. Problem description and mathematical model are explained in section 3. In section 4, we present a solution methodology. Section 5 describes the overview of fuzzy clustering. Lastly, section 6 and 7 define the numerical results and concluding marks, respectively.

2. Literature Review

Research in the area of PAS is addressed by formulating an integer linear programming model. It operates with limited clinical service capacity and a few numbers of chairs (Moreno and Blanco, 2018). Wang et al., (2018) proposed a dynamic programming model to choose a slot for patients and they used the idea of “complete set” to characterize optimal offer sets. Also, an approximate method for expressing the value of the system is presented. Heshmat et al. (2018) has used clustering and mathematical programming to improve the cancerous patient's appointment scheduling in chemotherapy clinics.
Hesaraki et al., (2018) generated an arrangement of vacant appointment slots and a template subject to the constraints of nurses. They used integer programming to solve the problem of balance between starting the first appointments and completing the last appointments.
Neglecting the influence of stochastic programming, a schedule may not be reasonable. Hence, Jiang et al. (2019) provided the outpatient scheduling problem considering unpunctuality by developing a stochastic programming model. Benders Decomposition combined with the Average Sample Approximation technique was utilized to minimize the weighted sum of all patient waiting times, doctor idle times, and overtime for Staff.
Moreover, a stochastic programming model has been developed to address the outpatient scheduling problem under unpunctuality (Jiang et al., 2017; Sedighpour et al., 2012). For example, Chen et al. (2018) formulated the stochastic optimization problem for appointment scheduling as its two-step deterministic equivalent to concurrently optimizing scheduling and overbooking arrangements to compensate patient no-shows with different time slot structures. Burdett et al. (2017) proposed the approach includes a mixed integer linear programming (MILP) model with an advanced extension. The maximum number of patients of each type, which can be treated within a time required to process a given cohort of patient, can be determined through the MILP models.

To improve the performance of an appointment system considering patient satisfaction and resource utilization, Srinivas and Ravindran (2018) proposed a prescriptive analytics framework. Dogru and Melouk (2018) developed an adaptive appointment scheduling model for treatment and proposed a simulation optimization approach to schedule appointments sequentially. Thus, desirable schedules have been provided from the perspective of patients and medical practices. Sevinc et al. (2013) developed a two-phase approach for solving the patient appointment scheduling problem. They proposed an adaptive negative-feedback scheduling algorithm for the first phase to control the system load. In the second phase, to assign patients to chairs, two heuristics based on the 'Multiple Knapsack Problem' have been compared.

A Markov decision process model has been proposed to optimize the appointments scheduling under patient priorities (Li et al., 2018). Marynissen and Demeulemeester (2019) have presented a review of multi-appointment scheduling for patients. To reduce the computation time in the performance evaluation of particular scheduling, Deceuninck et al. (2019) presented a variance reduction technique. Sauré et al. (2012) formulated a diminished infinite-horizon Markov decision process for scheduling cancer controls and treatments in chemotherapy units. The primary purpose of their model was to reduce waiting times with a cost-effective manner along with identifying the best policy for allocating available treatment capacity to demand. Alvarado and Ntaimo (2018) proposed the chemotherapy appointment scheduling under uncertainty, and they used the simulation and heuristics algorithms for solving the developed model. To reduce patient waiting time and personnel idle time in outpatient clinics, Anderson et al. (2015) developed an overlapping appointment scheduling model under stochastic service time. Also, they proposed a discrete-time Markov decision process model to increase server efficiency and improve customer service time.

For reserving and patient appointment in chemotherapy treatment, Condotta and Shakhlevich (2014) developed a multi-level optimization problem with a predetermined pattern for the appointment of patients. Work balance is of the imperative factor among nurses for not only the satisfaction of nurses, but also service delivery to patients. In most clinics, the assignment of nurses to patients relates to experience and past practices. Condotta and Shakhlevich (2014a) studied a multi-criteria optimization problem which appears in the context of booking chemotherapy appointments. The main feature of their model was to consider multiple appointments for each patient. Also, the nurse-patient assignment process in clinics is often a manual process (Acar and Butt, 2016; Dubyna et al., 2021). The Acuity-based nurse assignment was considered by Liang and Turkcan (2016).To reduce the computational time work, Deceuninck et al. (2019b) proposed a variance reduction technique to evaluate the performance of a specific schedule. Then, an evaluation approach is provided that involved a simulation optimization in giving insights into scheduling with unpunctual patients. Soltani et al. (2019) developed an appointment scheduling model to overcome the shortcoming considering stochastic service times along with customer no-shows for multiple-provider systems with equal providers.

2.1. Research gap

So far, models in PAS problem have been developed without regard to nursing skills. However, neglecting the nursing skills in PAS leads to the inappropriate allocation of nurses in terms of their skills. Also, if we use the model of Heshmat et al. (2018) for the PAS problem, we may not have an accurate understanding of the condition of the disease and the characteristics of the patients because of the complexity of the clustering algorithm,. The hard clusters for clustering patients like K-means clustering, applied in the model of Heshmat et al. (2018), considers only one aspect of the patient's features which in turn results in having a negative effect on the field of health and cancer. Furthermore, there is some complexity in the model which increases the computational time. Stemmed from the aforementioned gaps, the major novelties of this work are explained as follows:

- Using soft clustering techniques such as FCM Clustering so as to classify the nurses.
- Considering the nurses' skill factor in the allocation of each cluster to make the condition more realistic.
- Applying a hybrid approach based on Torabi-Hassini (TH) and Lagrange method to solve the proposed mathematical model.

3. Problem Description and Mathematical Model

In Figure 2, the concept of the proposed clustering method has been shown to consider outpatient patients that are planned to take the treatment in outpatient chemotherapy clinics. First, these patients are categorized (uncontrolled) and assigned into clusters. Each cluster is selected based on features such as treatment duration and type of cancer. Indeed, each circle represents a cluster in which the patients are grouped. The maximum time of treatment duration in each cluster is selected as the treatment time. To create clusters from given patients, a clustering algorithm has been implemented to cluster the patient mix. Second, the skills of nurses are identified by defining a cluster-skill matrix which determines the need of each cluster for each skill. Then, we develop a new the bi-objective mathematical programming model with considering the work-skill for nurses and priority factor for the PAS problem. The structure of constructing the proposed PAS problem has been shown in Figure 3.

In the following, the novel framework of the bi-objective mathematical model is presented. The aim of this bi-objective mathematical model is to minimize the treatment completion time as well as maximize the use of nurses' work-skills. The sets, parameters, and variables utilized to formulate the concerned problem are provided in Table 1.

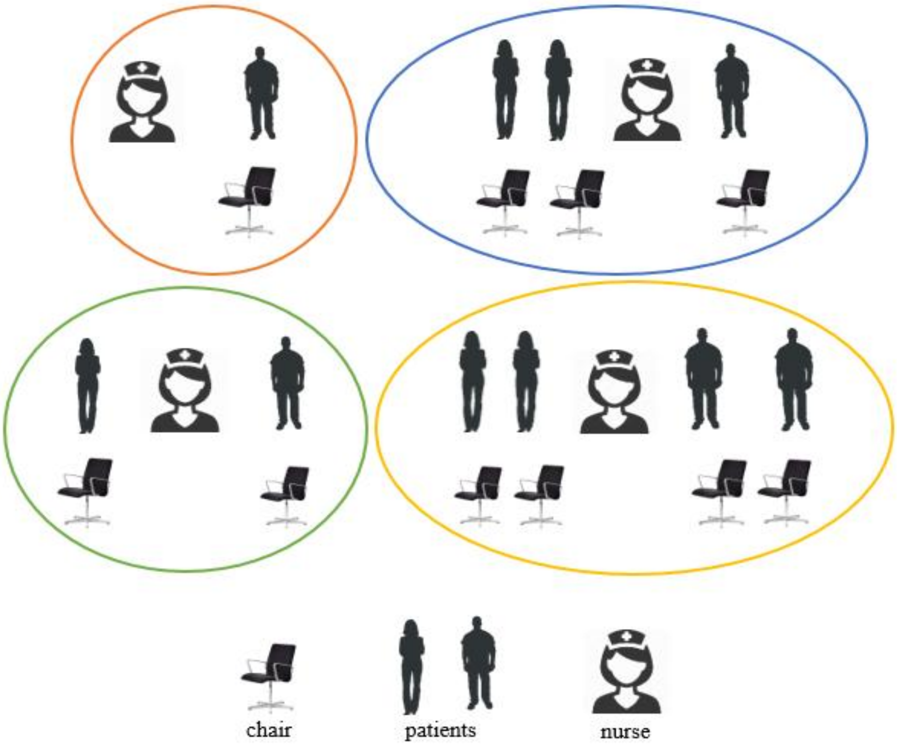

Fig. 1. Schematic for the proposed clustering method

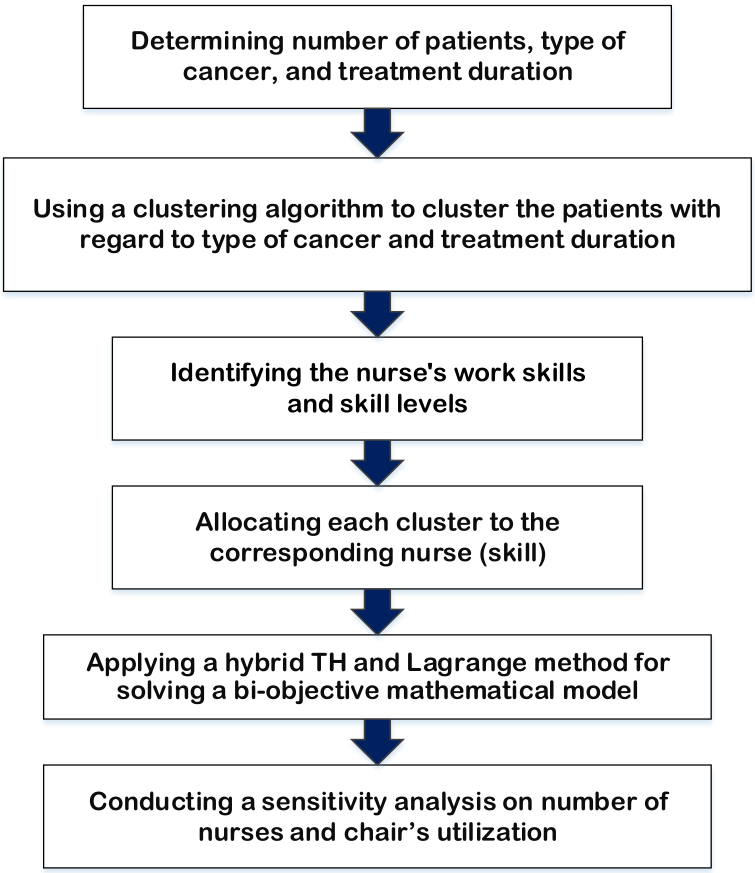

Fig. 3. The procedure for constructing Patient Appointment Scheduling problem

Table 1
Notation of the mathematical programming model.

Sets	**Description**
N	Number of nurses
B	Number of groups of chairs
S	Number of time slots
R_c	Treatment duration for a cluster of patient's *c* in time slut units
CT	Completion time of all treatments
x_{cjbsk}	Binary variable equals 1 if the treatment begins for cluster of patients c by nurse j with skill k on group of chairs b at a time slot s, and 0 otherwise
L	Number of clusters of patients
M_{jk}	Working time of nurse j with work skill k
W_{kj}	Priority factor (need of each cluster to each skill)
K	Number of work skill
y_{ck}	Binary parameter equals 1 if the work skill k assign to cluster of patients c, and 0 otherwise

$$\min C\,T \tag{1}$$

$$\max P = \sum y_{ck}\, W_{kj} \tag{2}$$

S.T.

$$\sum_{k=1}^{K}\sum_{j=1}^{N}\sum_{b=1}^{B}\sum_{s=1}^{s-R_c+1} x_{cjbsk} = 1 \qquad \forall\, c \in L \tag{3}$$

$$\sum_{k=1}^{K}\sum_{c\in L}\sum_{j=1}^{N}\sum_{u=max\{s-R_c+1.1\}}^{min\{s-R_c+1.s\}} x_{cjbuk} \le 1 \qquad \forall\, b = 1.\dots.B \quad \forall\, s = 1.\dots.S \tag{4}$$

$$\sum_{k=0}^{K}\sum_{c\in L}\sum_{B=1}^{B}\sum_{V=max\{s-R_c+1.1\}}^{min\{s-R_c+1.s\}} x_{cjbvk} \le 1 \qquad \forall\, j = 1.\dots.n \quad \forall\, s = 1.\dots.S \tag{5}$$

$$x_{cjbsk} \le y_{ck} \qquad \forall\, c \in L \;,\forall\, b \in B \;,\; \forall\, j \in N \;,\forall\, s \in S \;,\forall\, k \in K \tag{6}$$

$$\sum_{K}\sum_{C} x_{cjbsk} = \sum_{k} y_{ck} \qquad \forall\, b \in B \;,\; \forall\, j \in N \;,\forall\, s \in S \tag{7}$$

$$M_{jk} = \sum_{b=1}^{B}\sum_{s=1}^{s-R_c+1} x_{cjbsk}\, R_c \qquad \forall\, c \in L \;,\; \forall\, j \in N \;,\forall\, k \in K \tag{8}$$

$$M_{jk} \le CT \qquad \forall\, j \in N \;,\forall\, k \in K \tag{9}$$

$$x_{cjbsk} \in \{0.1\} \qquad \forall\, c \in L \;,\forall\, b \in B \;,\; \forall\, j \in N \;,\forall\, s \in S \;,\forall\, k \in K \tag{10}$$

The objective function (1) minimizes the total completion time of all the treatments. The objective function (2) maximizes the total priority factor for all clusters. Constraint (3) states that a cluster *c* among a set of clusters *L* is assigned to a nurse *j* with skill *k*, a group of chairs *b* in a time slot *s*. Constraint (4) guarantees that for each set of chairs, at most one cluster of patients is assigned. Constraint (5) ensures that each nurse with skill *k* is assigned to a single cluster of patients for each time slot and a group of chairs. Constraint (6) guarantees that each nurse with skill *k* assigns for one cluster. Constraint (7) ensures that skills assigned to clusters are in accordance with the cluster's requirements. Constraint (8) Calculates the working times for each assigned nurse with skill *k*. Constraint (9) guarantees that the calculated working times for each nurse with skill *k* is bounded by the total completion time, which is basically minimized by the objective function.

4. Solution Methodology

First of all, regarding the solution methodology, reception and registration of the patient profile should be done, which contains information about the set of patients and type of their cancer. Indeed, the type of cancer and treatment duration are of great importance among the required information. The treatment duration begins from the moment that the patient sits on the chair until the moment of leaves it. Second, Patients are clustering with FCM clustering algorithm optimally. Type of cancer and treatment duration are two features used for creating clusters of patients. These clusters are entered into the mathematical programming model as inputs. The third step relates to the nurse's work skills' identification, skill levels and need of each cluster to each skill. After developing the new PAS problem, we have used TH method to convert the bi-objective function to one (Torabi and Hassini, 2008). In the final step, we have solved the rendered mathematical programming model with the Lagrange method.

4.1. Hybrid TH and Lagrange method

The TH method was first provided by Torabi and Hassini (2008), which is one of the known approaches for dealing with multi-objective problems and obtaining the Pareto-optimal set (Ghodratnama et al., 2015).

In this method, taking into account the coefficients in the objective function, we can consider the optimistic state or pessimistic or between of those states.

For a problem with two objective functions, if Z_1 and Z_2 are respectively the value of the first and second objective functions, the steps of TH method will be as follows (Torabi and Hassini, 2008; Larki and Yousefikhoshbakht, 2014):

Step 1: Determine appropriate trapezoidal or triangular possibility distributions for the imprecise parameters and formulate the original model in the form of a fuzzy model.

Step 2: Convert the fuzzy objective functions to the deterministic type of them.

Step 3: Given the minimum acceptable possibility level for imprecise parameters, α, convert the fuzzy constraints into the corresponding crisp ones.

Step 4: Determine the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function by solving the corresponding MILP model in step 3 (Torabi and Hassini, 2008).

$$Z_1^{PIS} = \min Z_1 \ \ . \ \ Z_1^{NIS} = \max Z_1$$
$$Z_2^{PIS} = \min Z_2 \ \ . \ \ Z_2^{NIS} = \max Z_2$$

Step 5: Specify a linear membership function for each objective function as follows:

$$\mu_1(v) = \begin{cases} 1 & if \quad Z_1 < Z_1^{PIS} \\ \dfrac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}} & if \quad Z_1^{PIS} \le Z_1 \le Z_1^{NIS} \\ 0 & if \quad Z_1^{PIS} < Z_1 \end{cases}$$

$$\mu_2(v) = \begin{cases} 1 & if \quad Z_2 > Z_2^{PIS} \\ \dfrac{Z_2 - Z_2^{NIS}}{Z_2^{PIS} - Z_2^{NIS}} & if \quad Z_2^{NIS} \le Z_2 \le Z_2^{PIS} \\ 0 & if \quad Z_2 < Z_2^{NIS} \end{cases}$$

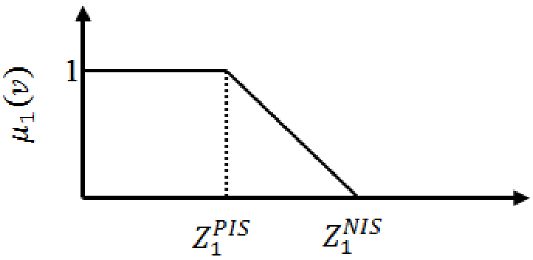

Fig. 4. Linear membership function for Z_1.

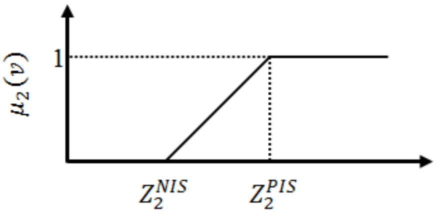

Fig. 5. Linear membership function for Z_2.

Step 6: Convert the bi-objective model into an equivalent single-objective using the following new crisp formulation.

$$\max \lambda(v) = \alpha\lambda_0 + (1-\alpha)\sum_h l_h\mu_h(v)$$
$$\text{subject to} \quad \lambda_0 < \mu_h(v) \quad . \quad h = 1.2$$
$$v \in F(v).\ \lambda_0$$
$$\alpha \in [0\ .\ 1]$$

where $\mu_h(v)$ denotes the satisfaction degree of h^{th} objective function and $\lambda_0 = \min_h\{\mu_h(v)\}$ denote the minimum satisfaction degree of objectives.

Step 7: With considering the loss factor α and the relative importance of the specified fuzzy objectives l_h, solve the deterministic single-objective model with the same restrictions.

After converting the bi-objective model to the single one, the Lagrange method is applied to solve the mathematical model. Indeed, in this method, violations of inequality in constraints will be considered as a penalty by using a Lagrange multiplier, which imposes a cost on infringements. These added costs are used instead of the strict inequality constraints in the optimization. In practice, this relaxed problem can often be solved more quickly than the original problem (Suhaimi et al., 2016).

5. Overview of Fuzzy Clustering

Data clustering and data mining are important issues. This is an unsupervised study where data of similar types are put into one cluster while data of another kind are put into a different cluster (Bora, 2014). Fuzzy clustering is a form of clustering in which each data point can belong to more than one clusters; in other words, in fuzzy clustering, data points can potentially belong to multiple clusters and data on the boundaries between several clusters are not forced to belong to one of the clusters fully. Whereas, in non-fuzzy clustering (hard clustering), data is divided into distinct clusters, where each data point can only belong to exactly one cluster. In fuzzy clustering methods, membership grades are assigned to each of the data points. Membership degrees are between 0 and 1 that indicating their partial membership. Clustering involves assigning data points to clusters such that items in the same cluster are as similar as possible, while items belonging to different clusters are as dissimilar as possible. Clusters are identified via similar features. These features in the medical may be is types of disease. Different similar features may be chosen based on the data or the application. Fuzzy clustering is more natural than hard clustering (such as k-Means) (Dunn, 1973).

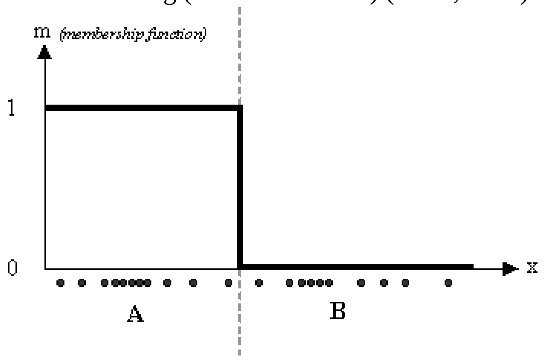

Fig. 6. Concept of membership function in the k-means clustering algorithm (Kumar, Andu, and Thanamani, 2013)

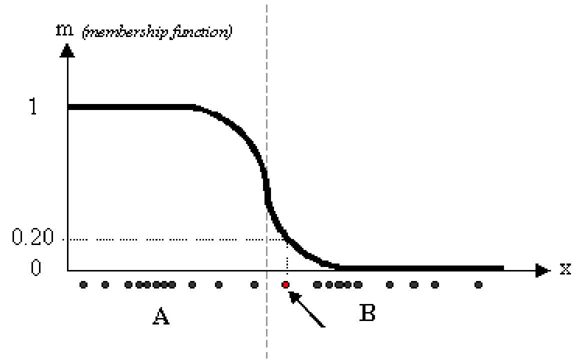

Fig. 7. Concept of membership function in the Fuzzy C-means clustering algorithm (Kumar et al., 2013).

In Figure 7, the datum shown as a red marked spot belongs more to the “cluster B” rather than the “cluster A”. The value 0.2 of ‘m’ indicates the degree of membership to “cluster A” for such datum.

5.1. Fuzzy C-Means clustering

FCM Clustering is a method of clustering which allows one piece of data to belong to two or more clusters. This method developed by Bezdek (2012) and improved, which is often used in pattern recognition. It is based on the minimization of the following objective function:

$$J_m = \sum_{i=1}^{N}\sum_{j=1}^{C} u_{ij}^m \left\|x_i - c_j\right\|^2 \quad . \quad 1 \le m < \infty$$

Where m is any real number greater than 1, u_{ij} is the degree of membership of x_i in the cluster j, x_i is the ith of d-dimensional measured data, c_j is the d-dimension center of the cluster, and ||*|| is any norm expressing the similarity between any measured data and the center.

Fuzzy partitioning is carried out through an iterative optimization of J_m, with the update of membership u_{ij} and the cluster centers c_j by:

$$u_{ij} = \frac{1}{\sum_{k=1}^{c}(\frac{\left\|x_i - c_j\right\|}{\left\|x_i - c_k\right\|})^{\frac{2}{m-1}}} \quad . \quad c_j = \frac{\sum_{i=1}^{N} u_{ij}^m x_i}{\sum_{i=1}^{N} u_{ij}^m}$$

This iteration will stop when, $max_{ij}\left\{\left|u_{ij}^{(k+1)} - u_{ij}^{(k)}\right|\right\} < \varepsilon$, where ε is a termination criterion between 0 and 1, whereas k are the iteration steps. This procedure converges to a local minimum or a saddle point of J_m.

The algorithm is composed of the following steps:

Step 1: Initialize $\mathrm{U} = [\mathrm{u_{ij}}]$ *matrix,* $\mathrm{U}^{(0)}$

Step 2: At k-step: calculate the centers vectors$\mathrm{C}^{(k)} = [\mathrm{c_j}]$ *with* $\mathrm{U}^{(k)}$

$$c_j = \frac{\sum_{i=1}^{N} u_{ij}^m x_i}{\sum_{i=1}^{N} u_{ij}^m}$$

Step 3: Update $\mathrm{U}^{(k)}.\ \mathrm{U}^{(k+1)}$

$$u_{ij} = \frac{1}{\sum_{k=1}^{c}(\frac{\left\|x_i - c_j\right\|}{\left\|x_i - c_k\right\|})^{\frac{2}{m-1}}}$$

Step 4: If $\left|u_{ij}^{(k+1)} - u_{ij}^{(k)}\right| < \varepsilon$ *, then STOP; otherwise return to step 2.*

Here, to explain more clearly, we consider the simple case of a mono-dimensional application of the FCM, with 20 data and 3 clusters, used for clustering. Figure 8 shows the membership value for each datum and each cluster. The overlapping shown in Figure 8 corresponds to fuzzy clustering.

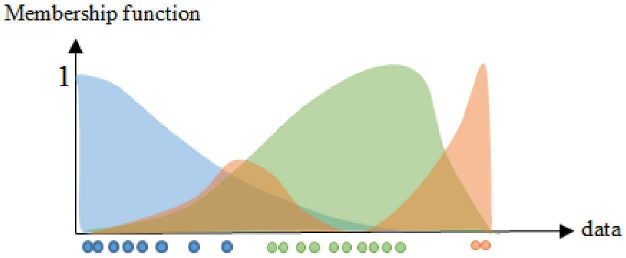

Fig. 8. Membership value for 20 data and 3 cluster

5.2. Using *Fuzzy C-means clustering*

In this study, to make the results close to reality, we used the FCM method to cluster the patients. We conducted this clustering according to data gathered from a chemotherapy clinic that includes 100 patients. This clustering has been implemented with considering the two characteristics including the type of cancer and the treatment duration of each patient. The performed clustering is shown in Figure 9.

As shown in Figure 9, 100 patients of a chemotherapy clinic, which refer to the clinic for receiving health care, have been clustered into seven clusters according to the two mentioned features. In this Figure, the "×" represents the clusters' centers.

6. Numerical Result (Case Study)

In this section, we used the real data of OCC in Tehran to evaluate the performance of the mathematical model and solution method. We also analyzed the performance of the rendered model in a variety of dimensions with a series of random data. After arriving the patients, we first determined the type of disease and the amount of time took to perform the treatment. Then, we used the FCM clustering algorithm to cluster the patient mix. Afterwards, we identified the nurse's work skills and skill levels as well as the need of each cluster for each skill that listed in table 2. Finally, using the problem information and the results of the previous sections, we solved the developed mathematical model. To do so, we first applied TH method to convert bi-objective function to the single one. Finally, we used the Lagrange method for solving the single-objective mathematical model.
Real data and results are presented in Tables 2-4:

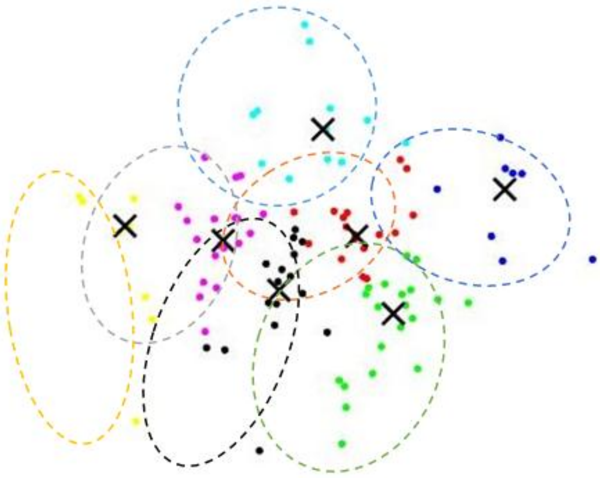

Fig. 9. Clustering 100 patients in a chemotherapy clinic

Table 2
Number of cluster, Treatment duration and need of each cluster to each skill.

cluster	$R_c(slot)$	y_{ck}					
		k=1	K=2	K=3	K=4	K=5	K=6
1	21	1	0	0	1	1	0
2	19	0	0	1	1	1	0
3	16	0	1	0	0	1	1
4	17	1	1	1	0	0	1
5	17	0	0	1	1	0	1
6	19	1	1	0	0	0	1
7	15	1	0	1	0	1	0

In this Table each slot is 15 minutes. The k-index denotes the skill, and y_{ck} represents the need for each cluster for each skill which is defined as 0 and 1.

Table 3
Work-skill of each nurse.

Work-skill	W_{kj}			
	j=1	j=2	j=3	j=4
1	0.3	0.2	0.25	0.25
2	0.2	0.3	0.35	0.15
3	0.4	0.2	0.2	0.2
4	0.35	0.25	0.25	0.15
5	0.15	0.25	0.2	0.4
6	0.6	0.5	0.8	0.5

The below results are obtained by solving the final mathematical programming model using the Lagrange method for the PAS problem based on the data in Tables 2-3 and the values of j = 4, b = 4 and the parameter TH is considered to be α = 0.5. This is the actual data for examining the model's performance in small dimensions in the environment of the chemotherapy clinic.

Table 4
Results of real data for PAS problem, (j=4, b=4).

CT (slot)	P
40.200	399.743

The results are obtained according to the data in Table 2-4 and the values of nurse number (j = 4) and chair number (b = 4), which indicates the maximum use of nurses' skills during a treatment period. This period includes 40.2 time slots, which is the minimum completion time of the treatment.

6.1. Analyzing the parameter TH

In this section, we examined the effect of the variations of the parameters of TH method on the objective functions. For this purpose, we considered five values 0.1, 0.3, 0.5, 0.7 and 0.9 for the TH parameter and compared the values of the objective functions with each other. The results obtained from these calculations are shown in Table 5.

Table 5
The effect of the variations of the parameter TH on the objective functions, (j=4, b=4).

parameter TH (α)	CT (slot)	P
0.1	86.655	676.375
0.3	64.224	482.097
0.5	40.200	399.743
0.7	37.33	325.554
0.9	35.87	299.870

As shown in the Table 5, by increasing the α, the first and second objective functions will get better and worse, respectively. That is, this parameter has a positive and negative correlation with the first and second objective functions, respectively.
To better understand the effect of the changes, we demonstrated the results in Figure 10.

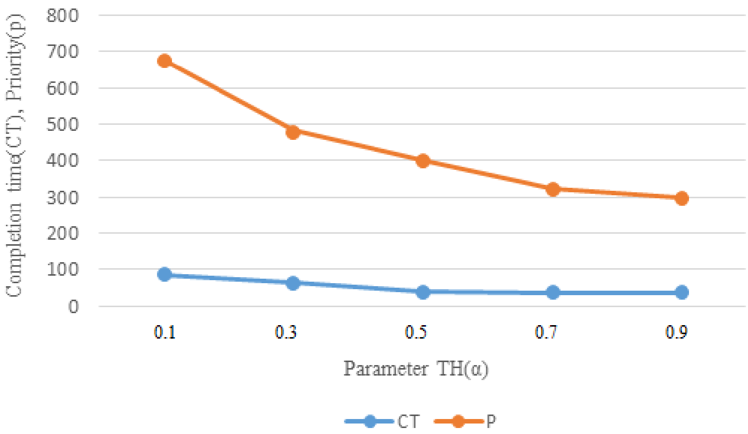

Fig. 10. The effect of the variations of the parameter TH (α) on the objective functions, (j=4, b=4).

6.2. The model solution for a randomly-generated data

In this section, data is randomly generated to analyze the developed model. For this purpose, we generate the data and, by making changes in them, we examine the sensitivity of each parameter with considering amount 0.5 for parameter TH.

6.2.1. Changes in the number of nurses

As shown in Figure 11, an increase in the number of nurses will improve the speed of treatment completion time and the rate of using nurses' skills. As a matter of fact, when the number of nurses and their range of skills increase, their allocation will be optimized and the speed of treatment will increase.

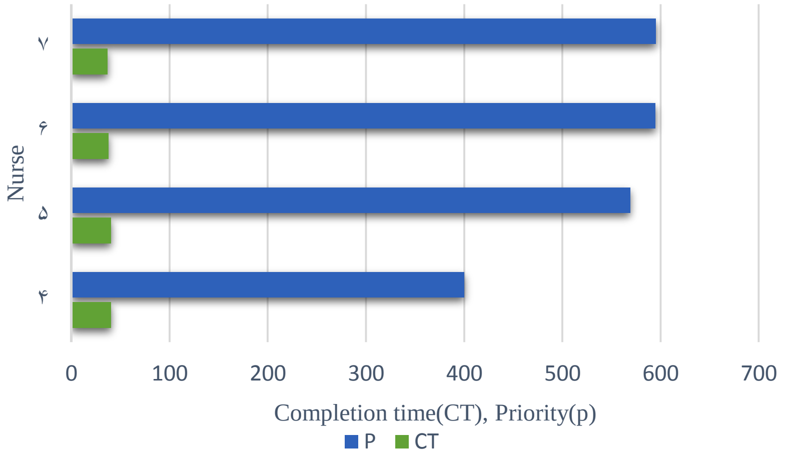

Fig. 11. The effect of changing the number of nurses on objective function

6.2.2 Changes in the number of chair group

As shown in Figure 12, an increase in the number of chairs leads to a decrease in the rate of using nurses' skills. The reason is that an increase in the number of chairs brings about an increase in the number of patients, who need receiving treatment. Therefore, given the constant number of nurses, inevitably, nurses with lower skills should be utilized. But this increase directly correlates with the completion time owing to the increased number of patients needed to received service.

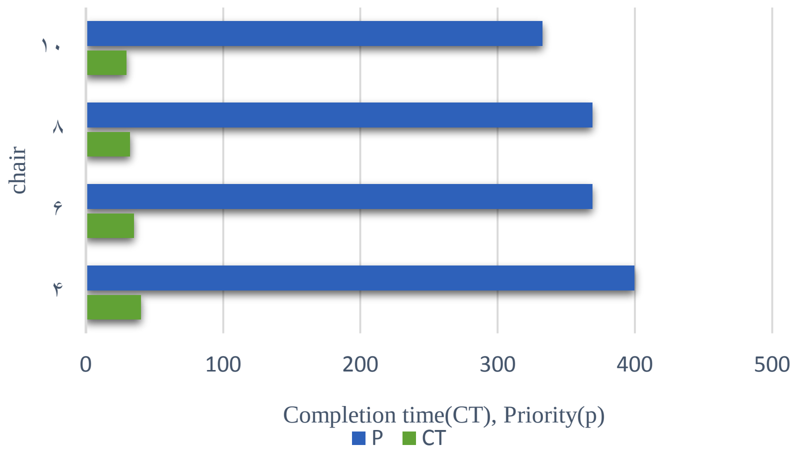

Fig. 12. The effect of chairs' utilization on objective functions

7. Conclusion

This paper provides a new approach to solve the Patient Appointment Scheduling (PAS) problem in Outpatient Chemotherapy Clinics (OCC). The developed mathematical model is inspired by cellular manufacturing. Such a strategy allows nurses in all periods to take advantage of clustering benefits. First, the patients have been categorized based on the cancer type and treatment duration. In doing so, the Fuzzy C-Means (FCM) clustering algorithm is applied to find the optimum clusters of the patient with considering the similar features of patients. Given the fact that the characteristics of patients are not definitively expressed or detected, we considered the fuzzy approach for clustering purpose. So far, none of the recent researches considered the allocation of nurses’ work skill along with patient clustering. Therefore, in this paper, we developed a bi-objective mathematical model for PAS considering nurses’ work skill and skill levels as well as the need of each cluster for each skill along with patients clustering. The aim of the rendered mathematical model is to minimize the completion of treatment and maximize the use of nurses' ability and skills. For solving the model, a hybrid approach based on TH method and Lagrange's analysis method is used the results represent the desirability of the proposed model. The results indicate that an increase in the number of nurses will enhance the treatment completion speed and allocation of nurses' work skill. On the other hand, an increase in the number of chairs in clinics will decrease the assignments of nurses' skills priority.

To further our research, we will use several features for clustering to make clusters more desirable. To provide a better service, we can add customer satisfaction as an essential factor to the model. Also, to improve the allocation of nurses, we can allocate them online and enable nurses to be displaced during treatment.

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